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ROTATING FLOWS OF KÁRMÁN-BÖDEWADT AND STAGNATION FLOWS

by

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ABSTRACT

The rotating flows of von Kármán and Bödewadt are re-examined on the basis of an extended boundary layer theory. It is found that the oscillating flow constructed by Bödewadt is due to underdetermined boundary data. A complete set of boundary data is defined for both problems in accordance with the practical flow models in consideration. The new solutions, which are defined as integrals of extended ordinary differential equations, eliminate the acknowledged discrepancies of the old solutions. Axisymmetric and plane stagnation flows with finite initial velocities, which are of significance for the design of piston machines, are constructed as special cases of von Kármán's problem.

FOREWORD

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Approved for Release:

/s/ R. H. LYDDANE
Technical Director

1. Introduction

The investigation [6] of the boundary layer along a flat surface produced by a potential vortex flow revealed an inconsistency of Prandtl's boundary layer theory which is negligible only for almost parallel boundary layers. This observation led to an important extension of the classical boundary layer theory which maintains the elliptic character of the flow problems considered. The new concept of the "limiting line of a boundary layer" yielded a powerful key to an asymptotic integration of the exact Navier-Stokes equations, which verified the extended boundary layer theory.

The new method of integration may now be applied to the boundary layer problems of von Kármán and Bödewadt [2, 3, 4, 5]. Although in both problems exact solutions of the Navier-Stokes equations have been constructed, the integrals obtained do not represent the motions desired in the practical applications. Bödewadt's solution describes a rotating flow over a flat plate which is assumed to be asymptotic to a solid-body vortex. However, the flow constructed possesses a source and sink distribution of infinite velocity at large distances from the axis of the vortex. These sources and sinks cause oscillations in the flow which are generally recognized to be meaningless in the practical applications. Since the vortex motion produced by a rotating disk in a fluid at rest (von Kármán's problem) satisfies the same differential equations and analogous boundary conditions as the flow generated by a solid-body vortex over a flat plate, Bödewadt's solution of the latter problem disqualifies also von Kármán's solution of the former problem. Indeed, von Kármán's solution possesses a sink distribution of infinite velocity at large distances from the axis of rotation. Because of the continuity of the flow, these sinks should require a source distribution of infinite velocity at large distances from the rotating disk. Furthermore, it may be mentioned that in both problems the pressure terms should be related to all velocity components (see equation (20)) and not only to one component. The pressure is determined mainly by the axial component at the axis and by the radial and tangential components far away from the axis.

It will be the subject of this paper to resolve the contradictions in the problems of von Kármán and Bödewadt. New asymptotic solutions will be defined as integrals of an extended set of ordinary differential equations, which are useful for practical applications. The new solution of von Kármán's problem yields as a special case the axisymmetric stagnation flow, which is produced by a homogeneous motion normal to a flat surface of infinite dimensions. A simple adjustment of the corresponding differential equations leads to the solution of the analogous problem in plane flow. The classical exact solutions of the Navier-Stokes equations describe, as is well known, the stagnation flows produced by nonhomogeneous motions of infinite initial velocities. The difference between the classical and the new stagnation flows displays the significance of both solutions in practical applications.

The new results and the results obtained in [6] demonstrate the strength of the unifying principle applied, which is manifested in the concept of the limiting line of a boundary layer. Furthermore, the results fully justify the extended boundary layer theory which was introduced in [6] as a replacement of the generally inconsistent classical boundary layer theory.

2. The Problems of von Kármán and Bödewadt

Let (u,v,w) denote the velocity vector of an axisymmetric vortex flow over a disk of infinite dimensions which is located at $z = 0$ in the cylindrical coordinate system (r,φ,z). The flow with constant kinematic viscosity ν, constant density ρ, and variable pressure p is governed by the Navier-Stokes equations

$$uu_r + wu_z - \frac{v^2}{r} = -\frac{1}{\rho} p_r + \nu\left[u_{rr} + \left(\frac{u}{r}\right)_r + u_{zz}\right] \tag{1}$$

$$uv_r + wv_z + \frac{uv}{r} = + \nu\left[v_{rr} + \left(\frac{v}{r}\right)_r + v_{zz}\right] \tag{2}$$

$$uw_r + ww_z = -\frac{1}{\rho} p_z + \nu\left[w_{rr} + \frac{1}{r} w_r + w_{zz}\right] \tag{3}$$

$$(ru)_r + (rw)_z = 0 \; . \tag{4}$$

In von Kármán's problem [5] the vortex motion is required to satisfy the boundary conditions

$$z = 0 \; : \; u = 0, \quad v = \omega r, \quad w = 0 \tag{5}$$

$$z = \infty \; : \; u = 0, \quad v = 0 \; . \tag{6}$$

In Bödewadt's problem [5] the vortex flow is required to fulfill the boundary conditions

$$z = 0 \quad : \quad u = 0,\ v = 0,\ w = 0 \tag{7}$$

$$z = \infty \quad : \quad u = 0,\ v = \omega r\ , \tag{8}$$

The angular velocity ω of the rotating disk in von Kármán's problem and of the rotating fluid in Bödewadt's problem is at one's disposal.

It is significant to mention that at this point von Kármán and Bödewadt failed to examine the uniqueness of the solutions of their problems. As Dirichlet's problem in potential theory shows, both problems yield families of solutions which differ in their singularities at $r = \infty$. Von Kármán and Bödewadt enforced the uniqueness for their problems by the well-known similarity assumptions. In return one is forced to accept the unrealistic source and sink distributions at $r = \infty$ which are the consequences of such drastic simplifications of the problems (see [6]).

In order to determine a solution of the Navier-Stokes equations which describes the vortex motion produced by a rotating disk in a fluid at rest, the following set of boundary conditions appears to be complete and adequate (see [6]):

$$z = 0 \quad : \quad u = 0,\ v = \omega r,\ w = 0 \tag{9}$$

$$z = \infty \quad : \quad u = 0,\ v = 0,\ w = w_\infty \tag{10}$$

$$\left.\begin{array}{l} z \geq 0 \\ r \to \infty \end{array}\right\} : \quad \frac{\partial u}{\partial r} \to 0,\ \frac{\partial v}{\partial r} \to 0,\ w \to w_\infty\ . \tag{11}$$

The solution of the Navier-Stokes equations, which describes the vortex motion produced by a fluid rotating as a solid body over a flat plate at rest, appears to be determined by the following set of boundary data:

$$z = 0 \quad : \quad u = 0, \quad v = 0, \quad w = 0 \tag{12}$$

$$z = \infty \quad : \quad u = 0, \quad v = \omega r, \; w = w_\infty \tag{13}$$

$$\left.\begin{array}{l} z > 0 \\ r \to \infty \end{array}\right\} : \quad \frac{u}{\omega r} \to 0, \quad \frac{v}{\omega r} \to 1, \; w \to w_\infty . \tag{14}$$

In both problems the constant w_∞ determines the strength of the corresponding singularity which is located at the point ($r = \infty$, $z = 0$).

Following the ideas of the derivations presented in [6], it is useful to introduce the limiting line $z = \delta(r)$ of the boundary layer adjacent to the surface $z = 0$. Provided a proper boundary layer exists, its limiting line $z = \delta(r)$ will then be characterized by the approximate data

$$z = \delta(r) \quad : \quad u \approx 0, \quad v \approx 0, \quad w \approx w_\infty \tag{15}$$

in von Kármán's case, and

$$z = \delta(r) \quad : \quad u \approx 0, \quad v \approx \omega r, \quad w \approx w_\infty \tag{16}$$

in Bödewadt's problem. The function $\delta(r)$ must be analytic and even. Thus it must yield an expansion of the form

$$z = \delta(r) \;=\; a - br^2 + \ldots \qquad \text{for } r < r_0 \; . \tag{17}$$

Because of the boundary data at $r = \infty$, $\delta(r)$ must decrease monotonically to zero as r tends to infinity.

Assuming tentatively that such a limiting line exists, then it is convenient to map the boundary layer $0 \le z \le \delta(r)$ onto the parallel strip $0 \le \zeta \le 1$ by the following similarity transformation:

$$r = r, \qquad \zeta = \frac{z}{\delta(r)} \; . \tag{18}$$

The conditions (9) through (16) suggest a search for solutions of the

Navier-Stokes equations which are of the special form

$$u = \omega rU(\zeta), \quad v = \omega rV(\zeta), \quad w = w_\infty W(\zeta) \tag{19}$$

$$\frac{p}{\rho} = -\frac{w_\infty^2}{2} P_0(\zeta) + \frac{\omega^2 r^2}{2} P_2(\zeta). \tag{20}$$

After carrying out these transformations one arrives at the following reduced Navier-Stokes equations:

$$(1 + \delta'^2\zeta^2)\ddot{U} - \left[\zeta(3\frac{\delta\delta'}{r} + \delta\delta'' - 2\delta'^2) + \frac{\omega\delta^2}{\nu}\left(\frac{w_\infty}{\omega\delta} W - \frac{r\delta'}{\delta}\zeta U\right)\right]\dot{U}$$
$$= \frac{\omega\delta^2}{\nu}\left[U^2 - V^2 + P_2 - \frac{r\delta'}{2\delta}\zeta\dot{P}_2 + \frac{w_\infty^2\delta'}{2\omega^2\delta r}\zeta\dot{P}_0\right] \tag{21}$$

$$(1 + \delta'^2\zeta^2)\ddot{V} - \left[\zeta(3\frac{\delta\delta'}{r} + \delta\delta'' - 2\delta'^2) + \frac{\omega\delta^2}{\nu}\left(\frac{w_\infty}{\omega\delta} W - \frac{r\delta'}{\delta}\zeta U\right)\right]\dot{V}$$
$$= 2\frac{\omega\delta^2}{\nu} UV \tag{22}$$

$$(1 + \delta'^2\zeta^2)\ddot{W} - \left[\zeta(\frac{\delta\delta'}{r} + \delta\delta'' - 2\delta'^2) + \frac{\omega\delta^2}{\nu}\left(\frac{w_\infty}{\omega\delta}W - \frac{r\delta'}{\delta}\zeta U\right)\right]\dot{W}$$
$$= \frac{1}{2}\frac{\omega\delta^2}{\nu}\left(\frac{\omega r^2}{w_\infty\delta}\dot{P}_2 - \frac{w_\infty}{\omega\delta}\dot{P}_0\right) \tag{23}$$

$$2U - \frac{r\delta'}{\delta}\zeta\dot{U} + \frac{w_\infty}{\omega\delta}\dot{W} = 0. \tag{24}$$

In these equations the primes and dots indicate the derivatives with respect to r and ζ. If second order terms in r are neglected, one arrives at the following set of ordinary differential equations by applying the expansion (17):

$$\ddot{U} + \left[8ab\zeta - \frac{aw_\infty}{\nu}W\right]\dot{U} = \frac{\omega a^2}{\nu}\left[U^2 - V^2 + P_2 - \frac{w_\infty^2 b}{\omega^2 a}\zeta\dot{P}_0\right] \tag{25}$$

$$\ddot{V} + \left[8ab\zeta - \frac{aw_\infty}{\nu}W\right]\dot{V} = 2\,\frac{\omega a^2}{\nu}UV \tag{26}$$

$$\ddot{W} + \left[4ab\zeta - \frac{aw_\infty}{\nu}W\right]\dot{W} = -\frac{aw_\infty}{2\nu}\dot{P}_0 \tag{27}$$

$$2U + \frac{w_\infty}{\omega a}\dot{W} = 0\ . \tag{28}$$

With the conventions

$$U = -\dot{G}(\zeta),\quad W = \frac{2\nu a}{w_\infty}G(\zeta),\quad P_0 = \frac{4\omega^2 a^2}{w_\infty^2}H(\zeta) \tag{29}$$

$$R = \frac{\omega}{\nu}\,\frac{a}{b}\ ,\ \sigma^2 = ab \tag{30}$$

one obtains the equivalent system of ordinary differential equations:

$$\dddot{G} + 2\sigma^2(4\zeta - RG)\ddot{G} + \sigma^2 R(\dot{G}^2 - V^2 + P_2 - 4\sigma^2\zeta\dot{H}) = 0 \tag{31}$$

$$\ddot{V} + 2\sigma^2(4\zeta - RG)\dot{V} + 2\sigma^2 R\dot{G}V = 0 \tag{32}$$

$$\ddot{G} + 2\sigma^2(2\zeta - RG)\dot{G} + \sigma^2 R\dot{H} = 0\ . \tag{33}$$

In these equations $G(\zeta)$ denotes the stream function of the continuity equation (28). The constant parameter R may be called the Reynolds number of the rotating motion. An appropriate justification for this notation will be given later.

An evaluation of the mapping (18) shows that the two singular points ($r = \pm \infty$, $z = 0$) are stretched into the corresponding lines ($r = \pm \infty$, $\zeta \geq 0$). Hence, in the (r, ζ) plane the exact motions considered are not determined by any boundary data at $r = \pm \infty$. Therefore all boundary data may be satisfied by solutions of the reduced equations (31), (32), and (33). This observation leads to the following solutions:

For small values of r and for $0 \leq \zeta \leq 1$, the functions $G(\zeta)$, $H(\zeta)$, $P_\zeta(\zeta)$, and $V(\zeta)$ represent approximate solutions of either von Kármán's problem or of Bödewadt's problem, provided:

(A) $G(\zeta)$ and $V(\zeta)$ are solutions of the differential equations

$$\dddot{G} + 2\sigma^2(6\zeta - RG)\ddot{G} + 8\sigma^4\zeta(2\zeta - RG)\dot{G} + \sigma^2 R(\dot{G}^2 - V^2 + P_\zeta) = 0 \tag{34}$$

$$\ddot{V} + 2\sigma^2(4\zeta - RG)\dot{V} + 2\sigma^2 R\dot{G}V = 0 \tag{35}$$

with either $P_\zeta(\zeta) \equiv 0$ and

$$\zeta = 0 : \quad G = 0, \ \dot{G} = 0, \quad V = 1 \tag{36}$$

$$\zeta = \infty : \quad G = G_\infty = \text{finite}, \ V = 0, \tag{37}$$

or with $P_\zeta(\zeta) \equiv 1$ and

$$\zeta = 0 \ : \ G = 0, \ \dot{G} = 0, \ V = 0 \tag{38}$$

$$\zeta = \infty \quad : \quad G = G_{\infty} = \text{finite}, \quad V = 1, \tag{39}$$

(B) $H(\zeta)$ is determined by

$$H = G^2 - \frac{\dot{G}}{R\sigma^2} + H_o - \frac{4}{R} \int_o^{\zeta} t\dot{G}(t)dt \ , \tag{40}$$

where

$$H_o = \frac{4}{R} \int_o^{\infty} t\dot{G}(t)dt, \tag{41}$$

(C) $G(\zeta)$ and $V(\zeta)$ satisfy the conditions

$$\zeta = 1 : \begin{cases} |G| \leq |G_{\infty}| - \epsilon, \ |\dot{G}| \leq \epsilon, \ |\ddot{G}| \leq \epsilon, \ |\dddot{G}| \leq \epsilon \\ |V| \leq 1 - \epsilon \end{cases} \tag{42}$$

for a common maximum ϵ ,

(D) the coefficients of the parabolic limiting line of the boundary layer

$$\delta(r) = a - br^2 \tag{43}$$

are determined by

$$a = \sigma \sqrt{\frac{\nu}{\omega}} R \qquad \text{and} \qquad b = \sigma \sqrt{\frac{\omega}{\nu}} \frac{1}{R} \tag{44}$$

where

$$w_{\infty} = 2\omega a G_{\infty} \ , \tag{45}$$

(E) the accuracy parameter ϵ is chosen sufficiently small.

The justification of this solution is based upon the same principles as the equivalent solution in [6]. An approximate solution of the Navier-Stokes equations within the boundary layer is joined with an exact solution outside the boundary layer in an almost analytic manner. The admitted violation of the analyticity along the limiting line of the boundary layer is controlled by the property (C). It may be emphasized that the property (C) does not confine the validity of the method applied to flows with large Reynolds numbers.

The existence of a solution to the remaining problem posed above can be illustrated in the same fashion as for the corresponding problem considered in [6]. A numerical method of solving the remaining boundary value problem will be presented in a paper in preparation. Simultaneously, numerical results will be displayed for various Reynolds numbers of both problems.

It is significant to point out the difference between the new equations (31) and (32) and von Kármán's and Bödewadt's equations. The new terms $8\sigma^2\zeta\ddot{G}$ and $8\sigma^2\zeta\dot{V}$ are obviously not negligible against the corresponding terms $2\sigma^2RG\ddot{G}$ and $2\sigma^2RG\dot{V}$ within the boundary layer $0 \leq \zeta \leq 1$, as they determine the character of the flow immediately at the surface $\zeta = 0$. It is also not useful to delete these terms outside the boundary layer because they guarantee the proper analytic continuations of the functions $G(\zeta)$, $\dot{G}(\zeta)$, $\ddot{G}(\zeta)$, $\dddot{G}(\zeta)$, and $V(\zeta)$ for $\zeta \geq 1$, as is required by property (A). These terms also confirm the conjecture formulated in [6] that the disturbance of the primary velocity must decay across the boundary layer with logarithmic order two, if a distinctive boundary layer is to be expected.

The appearance of the terms $8\sigma^2\zeta\ddot{G}$ and $8\sigma^2\zeta\dot{V}$ in the equations (31) and (32) resolve evidently the discrepancies in the solutions of von Kármán and Bödewadt, which were pointed out in the introduction. Their results are obtained by forcing the second coefficient b in the expansion (17) to zero, in order to attain the necessary uniqueness for their underdetermined problems. Exactly the same discrepancy arises by applying the classical boundary layer assumptions. Thus, the present problems fully justify the extended boundary layer theory which was introduced in [6].

It may be mentioned that the solutions constructed depend only upon the Reynolds number R at least up to the extent of the accuracy maintained by the equations (34) and (35). Thus all vortex flows with equal numbers R (see equation (30)) are similar to each other, which justifies the term Reynolds number for R. This statement follows through the introduction of the following new scales for all variables concerned:

$$G = \frac{g}{\sigma R}, \qquad V = \frac{f}{R}, \qquad \zeta = \frac{\eta}{\sigma} \tag{46}$$

$$H = \frac{h}{\sigma^2 R^2}, \qquad P_\zeta = \frac{h_\zeta}{R^2}. \tag{47}$$

One obtains the transformed system

$$\dddot{g} + 2(6\zeta - g)\ddot{g} + 8\zeta(2\zeta - g)\dot{g} + \dot{g}^2 - f^2 + h_\zeta = 0 \tag{48}$$

$$\ddot{f} + 2(4\eta - g)\dot{f} + 2\dot{g}f = 0 \quad , \tag{49}$$

which must be integrated under the conditions

$$h_2 \cong 0 \ , \quad \text{or } h_2 \cong R^2 \tag{50}$$

$$\eta = 0 : \quad g = 0 \ , \ \dot{g} = 0 \ , \quad f = R \ , \text{ or } f = 0 \tag{51}$$

$$\eta = \infty : \quad g = g_\infty = \text{finite}, \quad f = 0, \quad \text{or } f = R. \tag{52}$$

The similarity of two vortex flows with equal Reynolds numbers R is in full agreement with empirical observations. This significant coincidence indicates a certain relationship between the theoretical Reynolds number (30) and the well-known common Reynolds number. In order to discover this relationship it is helpful to examine the flow models of finite dimensions which led to the boundary data (9), (10), and (11), or (12), (13), and (14) of the approximate flow models of infinite dimensions considered.

In von Kármán's problem the vortex motion is produced by a rotating disk in a viscous fluid in a high cylinder at rest (see Figure 1). A special slit between the cylinder and the disk represents the sink which is located at ($r = \infty$, $z = 0$) in the model investigated. In Bödewadt's problem the configuration is the same, but the cylinder is rotating with the fluid over the disk at rest. Assuming that the friction forces along the cylinder are negligible, then the corresponding vortex motions are roughly represented by the solutions constructed above. If D denotes the diameter of the cylinder, then the common Reynolds number R_e is found to be

$$R_e = \frac{\omega}{\nu}\left(\frac{D}{2}\right)^2 . \tag{53}$$

An identification of the theoretical Reynolds number (30) with the common Reynolds number (53) leads to the equality

$$\frac{a}{b} = \left(\frac{D}{2}\right)^2 . \tag{54}$$

This interpretation of the ratio a/b appears plausible provided the accuracy of the solution constructed is sufficient to represent the entire flow for $0 \le r \le D/2$. In fact, the values $r = \pm\, a/b$ determine the intersections of the parabolic limiting line of the boundary layer $\delta(r)$ with the disk $z = 0$. (see equation (43) and Figure 1).

3. Stagnation Flows

The problem considered by von Kármán includes a stagnation flow, which is of interest in the design of piston machines. In order to display this special case it is useful to reconsider the flow model which is sketched in Figure 1. By holding the cylinder and the disk at rest and pressing a piston with the constant velocity w_∞ into the fluid, one produces a homogeneous flow which is disturbed by the outflow of the fluid through the slit between the disk and the cylinder.

If the diameter D of the cylinder is sufficiently large, then the stagnation flow described can be identified as a special case of von Kármán's problem investigated above. Indeed, by taking

$$v \equiv 0, \; V \equiv 0, \; f \equiv 0 \, , \tag{55}$$

and replacing ω by

$$\omega = w_\infty \sqrt{\frac{b}{a}} \tag{56}$$

in all corresponding differential equations and boundary data of the previous chapter, one arrives at the solution of the stagnation flow problem under consideration.

Let n = 1 denote the axisymmetric case and n = 0 the corresponding plane problem, then the equation (34) reduces to

$$\dddot{G} + \sigma^2[(8 + 4n)\zeta - 2^n RG]\ddot{G} + 2(n + 1)^2\sigma^4\zeta[2\zeta - RG]\dot{G} + \sigma^2 R\dot{G}^2 = 0 \tag{57}$$

which must be integrated under the boundary conditions

$$\zeta = 0 : \; G = 0, \; \dot{G} = 0 \, , \tag{58}$$

$$\zeta = \infty : \; G = \frac{-1}{2^n \sigma} = G_\infty \, . \tag{59}$$

For the derivation of equation (57) the following conventions have been used:

$$u = - w_\infty \sqrt{\frac{b}{a}} \, \dot{G}(\zeta), \; w = 2^n \sigma w_\infty G(\zeta), \; \frac{p}{\rho} = - 2^{2n-1} \sigma^2 w_\infty^2 H(\zeta) \tag{60}$$

$$z = \delta(r) = a - br^2 \, , \quad \zeta = \frac{z}{\delta(r)} \tag{61}$$

$$R = \frac{w_\infty}{\nu} \sqrt{\frac{a}{b}} \, , \quad \sigma^2 = ab \, . \tag{62}$$

The function $\delta(r)$ represents again the limiting line of the boundary layer adjacent to the disk $z = 0$. The dimensionless value R may be called the Reynolds number of the flow. The pressure is determined by the equation

$$H = 2^{n-1}G^2 - \frac{\dot{G}}{\sigma^2 R} + \frac{2(n+1)}{R} \int_{\zeta}^{\infty} t\dot{G}(t)dt \quad . \tag{63}$$

The solution of the remaining problem is, of course, acceptable only if the analogous condition (C) of the previous chapter is fulfilled. Numerical results have been obtained for various Reynolds numbers. They will be published in a paper in preparation.

The axisymmetric and plane stagnation flows investigated may be compared with the corresponding classical stagnation flows past a flat disk of infinite dimensions. In the present problems the fluid is kept within walls normal to the disk. Thus the initially homogeneous flow of finite velocity remains undisturbed almost throughout the entire fluid. The disturbance caused by the outflow is therefore limited to a thin boundary layer along the disk. In the classical stagnation flows the walls normal to the disk are removed (see Figure 2). This causes a nonhomogeneous potential flow past the disk which is disturbed by the friction forces along the surface of the disk. The steady state of this flow is maintained by an unbounded initial velocity at large distances from the plate. This type of stagnation flow represents evidently a model for viscous flows past blunt bodies with stagnation points.

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APPENDIX A

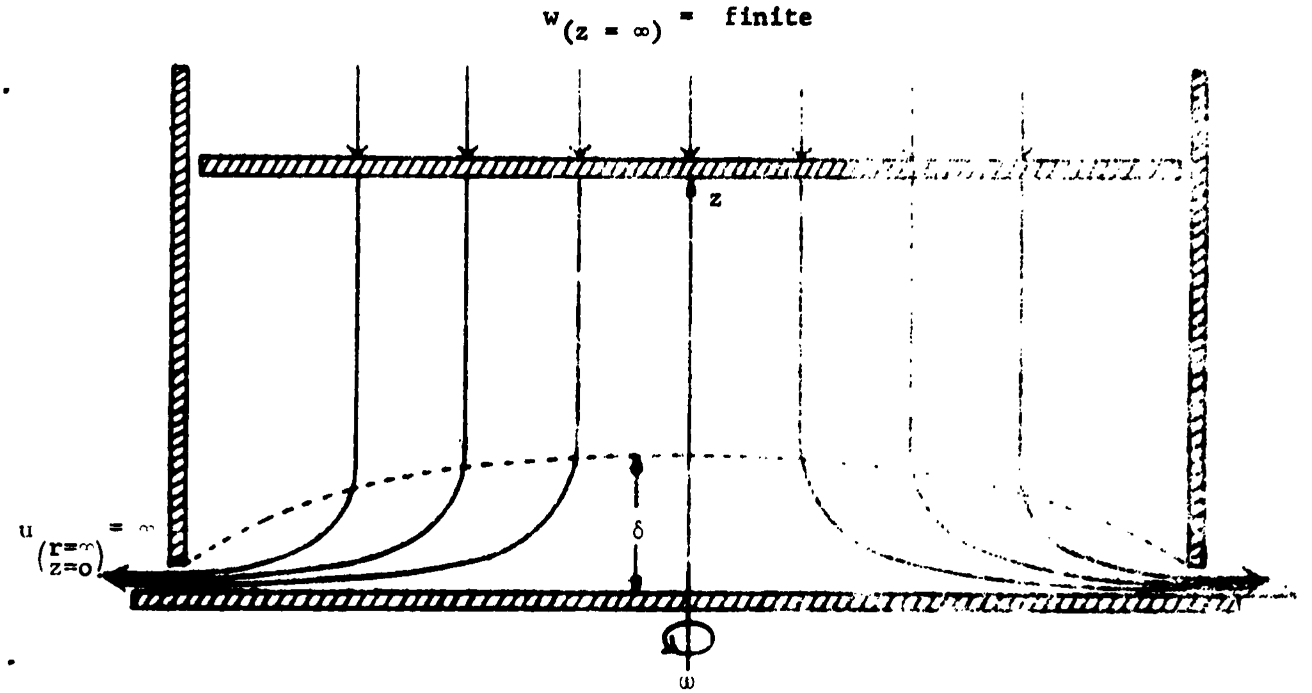

FIGURE 1 : MODEL OF THE PRESENT FLOW PROBLEMS
(Without piston for $\omega \neq 0$, with piston for $\omega = 0$)

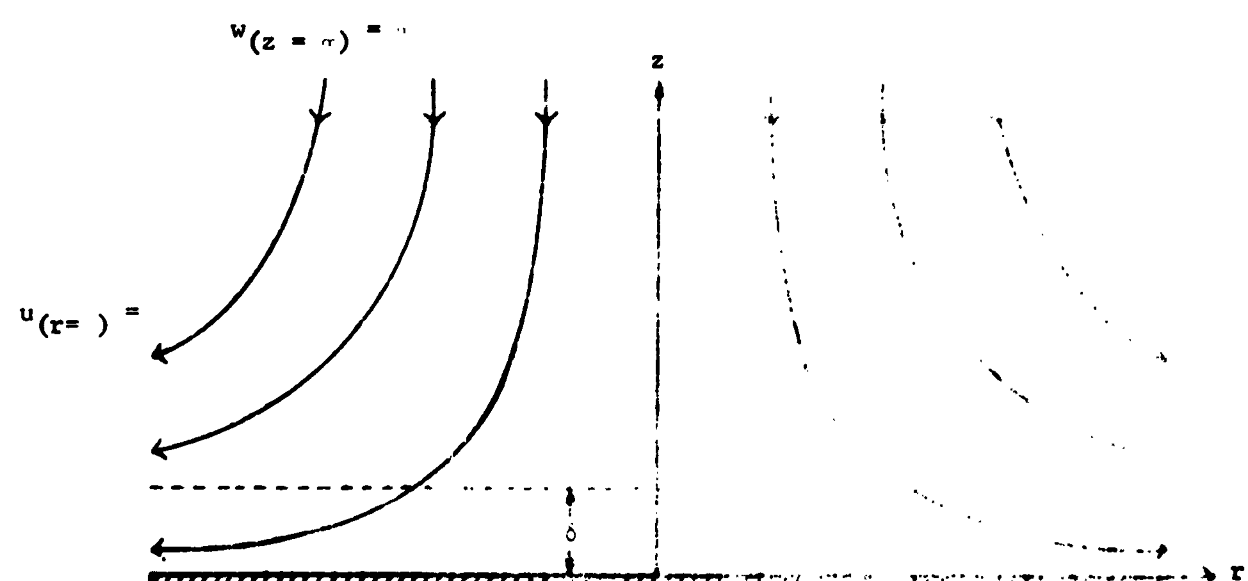

FIGURE 2 : MODEL OF THE CLASSICAL STAGNATION FLOW

APPENDIX B

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Boundary layer	BOUL	DATA	DATA		
Theory	THEY	Oscillating	OSCA		
Von Karman (name)	VONK	Pistons	PISN		
Models	MODE	Machinery	MACR		
Integral (Mathematics)	INTE	Finite (Mathematics)	FNIT		
Differential	DIFE	Initial	INIU		
Equations	EQUA	Velocity	VELC		
Axial	AXIA				
Symmetrical	SYMM				
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Problems	PRBL				

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ROTATING FLOWS OF KÁRMÁN-BÖDEWADT AND STAGNATION FLOWS, by E. W. Schwiderski and Hans Lugt. October 1962. 25 p.
UNCLASSIFIED

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1. Boundary layer - Theory
2. Axially symmetric flow - Velocity
3. Differential equations
4. Navier-Stokes equations
5. Piston machines - Design
6. Von Kármán, Theodore 1881 -
7. Bödewadt, U. T.
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